Technical Comments

Comment on "Thermally Induced Vibration and Flutter of a Flexible Boom"

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IN his recent paper,[1] Yu mentioned an equation and its solution. The equation is

$$\partial\kappa_T/\partial t + \kappa_T/\tau = K\cos(\alpha + \theta) \qquad (1)$$

The solution proposed by Yu is

$$\kappa_T = K\tau\cos\alpha - K\tau[\theta - \tau\dot{\theta} + \tau^2\ddot{\theta} - \tau^3\dddot{\theta} + \cdots]\sin\alpha \qquad (2)$$

If Eq. (2) is substituted in the left-hand side of Eq. (1), then for small values of θ

$$\begin{aligned}\partial\kappa_T/\partial t + \kappa_T/\tau &= -K\tau\dot{\alpha}\sin\alpha - K\tau[\theta - \tau\dot{\theta} + \tau^2\ddot{\theta} \\ &\quad - \tau^3\dddot{\theta} + \cdots]\dot{\alpha}\cos\alpha - K\tau[\dot{\theta} - \tau\ddot{\theta} \\ &\quad + \tau^2\dddot{\theta} - \tau^3\theta^{(iv)} + \cdots]\sin\alpha + K\cos\alpha \\ &\quad - K[\theta - \tau\dot{\theta} + \tau^2\ddot{\theta} - \tau^3\dddot{\theta} + \cdots]\sin\alpha \\ &= K[\cos\alpha - \theta\sin\alpha] - K\tau\dot{\alpha}[\sin\alpha \\ &\quad + (\theta - \tau\dot{\theta} + \tau^2\ddot{\theta} - \cdots)\cos\alpha] \\ &\cong K\cos(\alpha + \theta) - K\tau\dot{\alpha}[\sin\alpha + (\theta - \tau\dot{\theta} \\ &\quad + \tau^2\ddot{\theta} - \cdots)\cos\alpha] \qquad (3)\end{aligned}$$

Equation (3) matches Eq. (1) when

$$d\alpha/dt = 0$$

i.e., when the attitude of boom relative to sun radiation is constant.

A different solution series is proposed now which considers

$$d\alpha/dt = \omega \neq 0, \qquad d^2\alpha/dt^2 = 0 \qquad (4)$$

and τ, θ, $\dot{\theta}$ are small.

Let it be assumed that

$$\kappa_T = [k\tau/(1 + \tau^2\omega^2)^{1/2}][\cos(\alpha - \phi) - (\theta - \tau\dot{\theta} + \tau^2\ddot{\theta} - \cdots)\sin(\alpha - \phi)] \qquad (5)$$

where

$$\sin\phi = \tau\omega/(1 + \tau^2\omega^2)^{1/2}$$

and

$$\cos\phi = 1/(1 + \tau^2\omega^2)^{1/2}$$

Substituting Eq. (5) in the left-hand side of Eq. (1), and using the conditions set in Eq. (4), it is seen that

$$\begin{aligned}\frac{\partial\kappa_T}{\partial t} + \frac{\kappa_T}{\tau} &= -\frac{K\tau\omega}{(1 + \tau^2\omega^2)^{1/2}}\sin(\alpha - \phi) - \frac{K\tau}{(1 + \tau^2\omega^2)^{1/2}} \\ &\quad \times [\dot{\theta} - \tau\ddot{\theta} + \tau^2\dddot{\theta} - \cdots]\sin(\alpha - \phi) - \frac{K\omega\tau}{(1 + \tau^2\omega^2)^{1/2}} \\ &\quad \times [\theta - \tau\dot{\theta} + \tau^2\ddot{\theta} - \cdots]\cos(\alpha - \phi) + \frac{K}{(1 + \tau^2\omega^2)^{1/2}} \\ &\quad \times \cos(\alpha - \phi) - \frac{K}{(1 + \tau^2\omega^2)^{1/2}} \\ &\quad \times [\theta - \tau\dot{\theta} + \tau^2\ddot{\theta} - \cdots]\sin(\alpha - \phi) \\ &\cong \frac{K}{(1 + \tau^2\omega^2)^{1/2}}[\cos(\alpha - \phi) - \theta\sin(\alpha - \phi)] \\ &\quad - \frac{K\tau\omega}{(1 + \tau^2\omega^2)^{1/2}}[\sin(\alpha - \phi) + \theta\cos(\alpha - \phi)] \\ &\cong K\cos\phi \cdot \cos(\alpha + \theta - \phi) - K\sin\phi\sin(\alpha + \theta - \phi) \\ &= K\cos(\alpha + \theta)\end{aligned}$$

Thus, Eq. (5) provides a little more flexible solution to Eq. (1), but for a nominally three-axes stabilized spacecraft the difference between Eqs. (2) and (5) is small. So most of the observations of Yu remain valid.

Reference

[1] Yu, Y. Y., "Thermally Induced Vibration and Flutter of a Flexible Boom," *Journal of Spacecraft and Rockets*, Vol. 6, No. 8, Aug. 1969, pp. 902–910.

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Reply by Author to A. Das

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IT is noted correctly by Das that when α is constant and θ is small, Yu's original solution for κ_T is exact. Although Das has attempted to propose a solution for the case of α not equal to a constant, he has to approximate and replace the whole series

$$\theta - \tau\dot{\theta} + \tau^2\ddot{\theta} - \cdots$$

by θ in front of $\cos(\alpha - \phi)$ in the second line of his calculation below Eq. (5). Such an approximation, of course, is in conflict with and defeats the purpose of adopting the same series in the original assumed solution in Eq. (5).

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Index categories: Structural Dynamic Analysis; Structural Stability Analysis; Thermal Stresses.
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Errata

Formulating Propellants for Fully Case-Bonded End-Burning Motors

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IN the eighth line of the Contents section between of and 2,6- insert the following: "trimethylol propane, 1.05 equivalents of."

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Index categories: Solid and Hybrid Rocket Engines; Properties of Fuels and Propellants; Properties of Materials.